DHANAMANJURI UNIVERSITY

Examination - 2024 (June)

Four-Year Course BA/B.Sc. 4th Semester

Name of Programme	**:**	**B.A/B.Sc. Mathematics**
Paper Type	**:**	**6th**
Paper Type	**:**	**DSE-IV(Theory)**
Paper Code	**:**	**EMA-310**
Paper Title	**:**	**Mathematical Modelling and Graph Theory**
Full Marks	**:**	**100**
Pass Marks	**:**	**40**
Duration	**:**	**3 Hours**

The figures in the margin indicate full marks for the questions.
Question no. 1 is compulsory and choose any six questions from the remaining part:

1. Answer the following questions: **$5 \times 2 = 10$**

(a) What is a tree? Draw the eleven unlabeled trees with seven vertices.

(b) Write a short note on Monte-Carlo Simulation? What are the advantages of using Simulation?

2. Solve by using dual simplex method for the following LPP **15**

Minimize $Z = 5x_1 + 6x_2$
Subject to:

$$x_1 + x_2 \geq 2$$
$$4x_1 + x_2 \geq 4$$
$$x_1 \geq 0, x_2 \geq 0$$

3. Consider the following LPP **15**

Maximize $Z = 35x_1 + 50x_2$
Subject to:

$$4x_1 + 6x_2 \leq 120$$
$$x_1 + x_2 \leq 20$$
$$2x_1 + 3x_2 \leq 40$$
$$x_1 \geq 0, x_2 \geq 0$$

If the RHS of constraint changes from $[120, 20, 40]^T$ to $[75, 15, 50]^T$, find the new optimal solution.

4. Solve the following LPP

Maximize $Z = 3x_1 + 4x_2 + 6x_3 + 10x_4$
Subject to:

$$x_1 + x_2 + x_3 + x_4 \leq 12$$
$$2x_1 + 4x_2 + 9x_3 + 10x_4 \leq 70$$
$$x_1, x_2, x_3, x_4 \geq 0$$

And compute the range of a_{23} so that the optimal solution remains optimal.

5. Solve the following LPP **15**

Max $Z = 4x_1 + 6x_2 + 2x_3$
Subject to:

$$x_1 + x_2 + x_3 \leq 3$$
$$x_1 + 4x_2 + 7x_3 \leq 9$$
$$x_1, x_2, x_3 \geq 0$$

Find the range of basic variable coefficient c_1 such that the current optimal remains optimal.

6. **15**

A small project is composed of activities whose time estimate is listed in the table below. Activities are identified by their beginning and ending node numbers.

Activity (i-j)	Optimistic Time	Most likely Time	Pessimistic Time
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

i) Draw the project network.

ii) Find the expected duration and variance for each activity. What is the expected project length?

iii) Calculate the variance and standard deviation of the project length. What is the probability that the project will be completed:

a) at least 4 weeks earlier than expected?

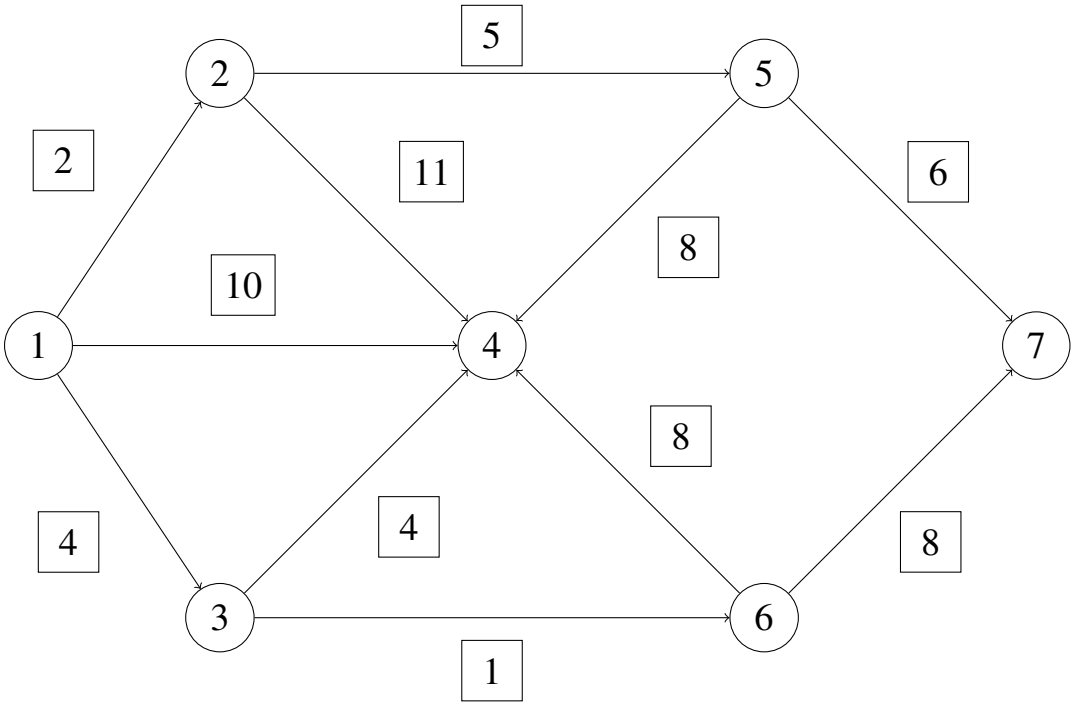
b) no more than 4 weeks later than expected time?

7. Answer the following questions: 10 + 5 = 15

(a) Dr. Strung is a dentist. He gives appointments to patients every half an hour. However, he does not know the nature of illness of patients arriving at his clinic. From past records, he has the following probability distribution and also knows the exact treatment timings. He starts his clinic at 8:00 A.M. Using the following information determine the average waiting time of the customers and idle time of the doctor. Use the random numbers 76, 60, 46, 86, 87, 68, 17, 22, 05, 25 and the table depicts the probability distribution with respect to past records:

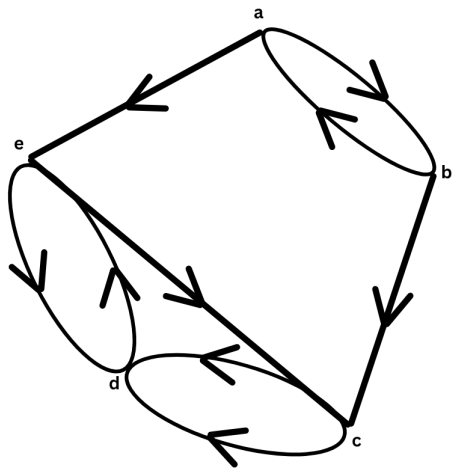
Nature of illness	**Probability**	**Time taken for treatment (min)**
Filling	0.10	50
Check up	0.30	15
Crowning	0.15	40
Cleaning	0.30	15
Extraction	0.15	30

(b) The network below gives the permissible routes and their lengths in miles between stations of city 1 (node 1) and six other cities (nodes 2-7). Determine the shortest route and hence the shortest distance from city 1 to city 7.



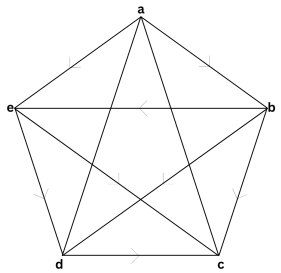
8. Answer the following questions:

(a) What is Adjacency matrix of a digraph? Consider the digraph below:



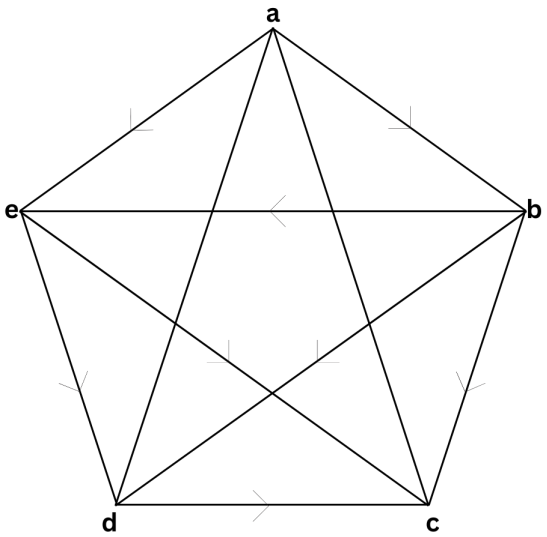
Write down the adjacency matrix A. Calculate A^2, A^3 and A^4 and hence find the number of walks of length 1, 2, 3, and 4 from vertex b to vertex d. Are there walks of length 1, 2, 3, and 4 from vertex d to vertex b?

(b) Find the matrix B of the given digraph. And hence determine whether the digraph is strongly connected or not.



9. Answer the following questions:

(a) Define Eulerian and Hamiltonian Digraph. In the digraph given below:



a) Find a cycle of length 3, 4, and 5.

b) Find a Eulerian trail.

c) Find a Hamiltonian cycle.

(b) What is a complete bipartite graph? Draw $K_{3,4}$ and $K_{1,7}$ and how many vertices and edges does each graph have? Show that the maximum number of edges in a complete bipartite graph of n vertices is $\frac{n^2}{4}$.