DHANAMANJURI UNIVERSITY

Examination- 2024 (Dec)

Four year course B.Sc./B.A. 5^{th} Semester

Name of Programme : B.Sc./B.A. Mathematics(Honours)
Paper Type : CORE XIII{Theory}
Paper Code : CMA-313
Paper Title : Multivariate Calculus
Full Marks : 80
Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions:
Answer all the question.

1. Choose the correct answer from the following and rewrite: **$1 \times 3 = 3$**

a) If $x = rcos\theta$ and $y = rsin\theta$, then the value of $\frac{d\theta}{dx}$ is
i) $\frac{x}{x^2+y^2}$ ii) $-\frac{x}{x^2+y^2}$
iii) $-\frac{y}{x^2+y^2}$ iv)$\frac{y}{x^2+y^2}$

b) If $x = rsin\theta cos\phi, y = rsin\theta sin\phi$ and $z = rcos\theta$,then
$\int\int\int f(x, y, z)dxdydz$
i) $\int\int\int f(r, \theta, \phi)r^2 sin\phi drd\theta d\phi.$
ii) $\int\int\int f(r, \theta, \phi)r^2 cos\phi drd\theta d\phi.$
iii) $\int\int\int f(r, \theta, \phi)r^2 cos\theta drd\theta d\phi.$
iv) $\int\int\int f(r, \theta, \phi)r^2 sin\theta drd\theta d\phi.$

c) The Stoke's Theorem is a relation between
i) line integral and double integral.
ii) line integral and surface integral.
iii) line integral and volume integral.
iv) surface integral and volume integral.

2. Write very short answer for each of the following questions: $1 \times 6 = 6$

a) Find $\frac{\partial^2 x}{\partial x \partial y}$, where $z = x^2 + 2x^2y^2 + y^2$.

b) Define gradient of a scalar function.

c) Find $\int_0^2 \int_0^{x^2} e^{\frac{2}{x}} dydx$.

d) If(x,y,z)and (z, ρ, ϕ) are the cartesian and cylindrical coordinates of a point P, then write the value of $dxdydz$ in terms of cylindrical coordinates.

e) If the vector $\overrightarrow{V} = (x + 3y)\hat{\imath} + (y - 2z)\hat{\jmath} + (x + az)\hat{k}$ is solenoidal, then find the value of a.

f) Find the work done by a force $y\hat{\imath} - x\hat{\jmath}$ which displaces a particle from origin to the point $(\hat{\imath} + \hat{\jmath})$.

3. Write short answer for each of the following questions: 3×5=15

a) Prove that $Y = f(x + at) + g(x - at)$ satisfies $\frac{\partial^2 y}{\partial t^2} = a^2(\frac{\partial^2 y}{\partial x^2})$,where f and are assumed to be at least twice differentiable and a is any constant.

b) If $\phi = 3x^2y - y^3z^2$, find grad ϕ at the point (1,-2,-1).

c) Evaluate $\int \int_R xydxdy$, where R is the quadrant of the circle $x^2 + y^2 = a^2$ where $x \geq 0, y \geq 0$.

d) Find the volume of the solid bounded by the parabolic $y^2 + z^2 = 4x$ and the plane $x = 5$.

e) A vector field is given by $\overrightarrow{F} = (2y + 3)\hat{\imath} + xz\hat{\jmath} + (yz - x)\hat{k}$.Evaluate $\int_c \overrightarrow{F}.d\overrightarrow{r}$ along the path C
where $x = 2t, y = t, z = t^3$ from t=0 to t=1.

4. Answer any four of the following questions: 4×5=20

a) By using $\epsilon - \delta$ definition, prove that $\lim\limits_{(x,y)\to(2,1)} (3x + 2y) = 8$.

b) Find teh equations of the target plane and the normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point (2,1,-3).

c) Evaluate $\displaystyle\int_0^{2a}\int_0^{\sqrt{(2a-x^2)}} x^2 dydx$.

d) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ and the cylinder $x^2 + y^2 = a^2$.

e) Prove that $(y^2 - z^2 + 3yz - 3x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.

5. Answer any two of the following questions: 6×2=12

a) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$,then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

b) Examine $f(x, y) = x^3 + y^3 - 3axy$ for maximum and minimum values.

c) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

6. Answer any two of the following questions: 6×2=12

a) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^y cos2y\sqrt{(1 - a^2sin^2x)}dxdy$.

b) Change the order of integration and evaluate $\int_0^a \int_0^y \frac{dxdy}{\sqrt{\{(a^2+x^2)(a-y)(y-x)\}}}$.

c) Evaluate $\displaystyle\int\int\int_R (x^2 + y^2 + z^2)dxdydz$ where R denotes the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = a(a > 0)$.

7. Answer any two of the following questions: 6×2=12

a) Evaluate $\int\int_s \overrightarrow{A}.\hat{n}ds$ where $\overrightarrow{A} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

b) Using Stoke's Theorem, evaluate $\int_c [(2x - y)dx - yz^2 dy - y^2 z dz]$,Where C is the circle $x^2 + y^2 = 1$, corresponding to the surface of unit radius.

c) Apply Guass Theorem to evaluate $\int\int_s \overrightarrow{F}.\hat{n}ds$ where $\overrightarrow{F} = 4x^3\hat{i} - x^2y\hat{j} + x^2zk$ and S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by the planes $z = 0$ and $z = b$.

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