DHANAMANJURI UNIVERSITY

Examination- 2023 (Dec)

Three year course B.Sc. 5th Semester

Name of Programme	**:**	**B.A/B.Sc. Mathematics (Honours)**
Semester	**:**	**5th**
Paper Type	**:**	**Core-XI (Theory)**
Paper Code	**:**	**CMA-311**
Paper Title	**:**	**Multivariate Calculus**

Full Marks : 100

Pass Marks : 40 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions

Answer all the questions:

1. Choose the correct answer from the following and rewrite it: $\mathbf{1 \times 6 = 6}$

a) Iterated limits are called

i) double limit ii) repeated limit

iii) both A and B iv) neither A nor B

b) The function f(x,y) is said to be continuous in a domain D if it is continuous at

i) one point of D ii) at each point of D

iii) at least two point of D iv) Both A and B

c) The condition $\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0, \frac{\partial f}{\partial z} = 0$ for maximum or minimum of a function is

i) necessary condition ii) Sufficient Condition

iii) A and B both iv) neither A nor B

d) The value of $\int_{a^2}^{a} \int_0^{\sqrt{a^2-y^2}} dxdy$ is

i) $\frac{\pi a^2}{4}$ ii) $\frac{\pi a^2}{8}$

iii) $\frac{\pi a^2}{12}$ iv) $\frac{\pi a^2}{16}$

e) A necessary and sufficient condition that the line integral $\int_c \vec{A} . \overrightarrow{dr} = 0$ for every closed curve C is that

i) Div $\vec{A} = 0$ ii) Cur $\vec{A} = 0$

iii) div $\vec{A} \neq 0$ iv) cur $\vec{A} \neq 0$

f) If $\vec{F}$ is the velocity of a flux particle then $\int_c \vec{F} \overrightarrow{dr}$ represents

i) Circulation ii) Work done

iii) Flux iv) Conservative field

2. Write very short answer for each of the following questions: $\mathbf{1 \times 10 = 10}$

a) Define limit of a function of two variables.

b) If f(x,y)= $x^3y - xy^3$, find $\left(\frac{1}{\frac{\partial f}{\partial x}} + \frac{1}{\frac{\partial f}{\partial y}}\right)_{\substack{x=1\\y=2}}$

c) Write the expression of total differential for the function $z = f(x, y)$.

d) If $\emptyset = 3x^2y - y^3z^2$, find grad $\emptyset$.

e) Find the stationary points of f(x, y) = $x^2 + y^2 + 6x + 12$.

f) Evaluate $\int_0^a \int_0^{\sqrt{ay}} xy\, dx\, dy$.

g) Write the double integral $\iint f(x, y) dx\, dy$ into polar coordinates.

h) Evaluate $\int_{-3}^{3} \int_0^1 \int_{-1}^{2} dxdydz$.

i) Define line integral of a vector function.

j) Write the formula for Stoke's theorem.

3. Write short answer for each of the following question: $\mathbf{3 \times 12 = 36}$

a) Let f(x,y)= $\frac{x^2y}{x^4+y^2}$, $x^4 + y^2 \neq 0$ and $f(0,0) = 0$, show that $\lim_{(X,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ does not exist.

b) Show that $f(x, y) = \begin{cases} \frac{xy^3}{x^2+y^6}, & x \neq 0, y \neq 0 \\ 0, & x = 0, y = 0 \end{cases}$ is not continuous at (0,0).

c) If u= sin $\frac{x}{y}$ + log$\frac{y}{x}$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ = 0.

d) Let z=$x^2 + y^2$ where x=$\frac{1}{t}$, y=t^2, Find $\frac{dz}{dt}$ in two ways

i) by first expressing z explicitly in terms of t.

ii) by using chain rule

e) Find the directional derivatives of f(x,y)=3-2$x^2 + y^3$ at the point (1,2) in the direction of the unit vector $\vec{u} = \frac{1}{2}\vec{\imath} - \frac{\sqrt{3}}{2}\vec{\jmath}$

f) Find a unit normal to the surface $x^2 - y^2 + z^2 = 3$ at (1,-1,1)

g) Let T be the triangular region enclosed by the lines $y = 0$, $y = 2x$and $x = 1$, then evaluate $\iint_T (x + y)\, dxdy$ using an iterated integral with

i) y integration first and

ii) x integration first

h) Find the volume of the solid bounded above by the plane z=y and below in the xy-plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant, where $f(x, y) = y$.

i) Evaluate $\iiint_B z^2ye^x$, where B is the box given by 0$\leq x \leq 1, 1 \leq y \leq 2, -1 \leq z \leq 1$

j) Evaluate the integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2) dxdy$ by changing into polar coordinates.

k) Find the work done where a force $\vec{F} = (x^2 - y^2 + x)\vec{\imath} - (2xy + y)\vec{\imath}$ moves a particle from origin to (1,1) along a parabola $y^2 = x$.

l) Show that $\vec{F} = yz\vec{\imath} - xz\vec{\jmath} + xy\vec{k}$is conservative and find a scalar potential function.

4. Answer any two questions from the following questions: $\mathbf{6 \times 2 = 12}$

a) Find the equations of the tangent plane and normal line to the surface z=$x^2 - y^2$ at the point (1,1,0).

b) If V= f (x-y, y-z, z-x), then prove that $\frac{\partial V}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial V}{\partial z}=0.$

c) Let f(x,y) = $\tan^{-1}\frac{y}{x}$, then find the directional derivative of f at (1,2) in the direction that makes an angle of $\frac{\pi}{3}$ with the positive x-axis.

5. Answer any two questions from the following questions: $\mathbf{6 \times 2 = 12}$

a) Find the point on the plane x+2y+z=5 that is closed to the point $(0,3,4)$.

b) By using Lagrange method, find the maximum and minimum values of $\mathrm{f(x,y,z) = x - y - z}$ on the sphere $x^2 + y^2 + z^2 = 100$.

c) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{\imath} + (3xz - 2xy)\vec{\jmath} + (3xy - 2xz + 2z)\vec{k}$ is both solenoidal and irrotational.

6. Answer any two questions from the following: $\mathbf{6 \times 2 = 12}$

a) Evaluate: $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\,dydx$ by converting to polar co ordinates.

b) Evaluate: $\iiint_R (x^2+y^2+z^2)\,dxdydz$, where R denotes the region bounded by $\mathrm{x = 0, y = 0, z = 0}$ and x+y+z=a ,a>0.

c) Find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half cone $z = \sqrt{x^2+y^2}$ and the xy- plane.

7. Answer any two questions from the following: $\mathbf{6 \times 2 = 12}$

a) Verify Green's Theorem in the plane $\oint_C [(xy+y^2)dx + x^2dy]$, where C is the closed curve of the region bounded by y=x and y=x^2.

b) Evaluate: $\iint_S \vec{A}.\hat{n}ds$ where $\overrightarrow{A} = 18z\vec{\imath} - 12x\vec{\jmath} + 3y\vec{k}$ and S is the part of the plane 2x+3y+6z=12 included in the first octant.

c) Evaluate: $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ and S is the surface of the cube bounded by x=0,x=1,y=0,y=1,z=0,z=1 by using the Divergence Theorem.

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