DHANAMANJURI UNIVERSITY

Examination - 2024 (June)

Four-Year Course BA/B.Sc. 4th Semester

Name of Programme	**:**	**B.A/B.Sc. Mathematics**
Paper Type	**:**	**Core-XII(Theory)**
Paper Code	**:**	**CMA-212**
Paper Title	**:**	**Numerical Analysis**
Full Marks	**:**	**40**
Pass Marks	**:**	**16**
Duration	**:**	**2 Hours**

The figures in the margin indicate full marks for the questions
Answer the following questions:

1. Choose and rewrite the correct answer for each of the following questions: **$1 \times 3 = 3$**

a) The value of $\frac{\Delta x^{(n)}}{\Delta x}$ is

i) $n!h^n$
ii) nx^{n-1}
iii) nhx^{n-1}
iv) $n!x^{n-1}$

b) Simpson's one-third rule can be represented by equation of a

i) Circle
ii) Straight line
iii) Parabola
iv) Hyperbola

c) The rate of convergence of $\lim_{n\to\infty} \frac{n+3}{n+7} = 1$ is

i) $O\left(\frac{1}{n}\right)$

ii) $O\left(\frac{1}{n^2}\right)$

iii) $O\left(\frac{1}{2^n}\right)$

iv) $O\left(\frac{1}{2n}\right)$

2. Write very short answer on any two from the following questions: $1 \times 2 = 2$

a) Show that $(\Delta - \nabla) \cong \Delta\nabla$.

b) Evaluate $\Delta^2(3e^x)$.

c) Define Numerical Integration.

d) State Runge-Kutta method of 2nd order.

e) Give the iteration scheme of Regula-falsi method.

f) Define Algorithm.

3. Write short answer on any three from the following questions: $3 \times 3 = 9$

a) Find $\Delta\left[\frac{2^x}{(x+1)}\right]$, $h = 1$.

b) Estimate the missing term in the following table:

x	0	1	2	3	4
$f(x)$	1	3	9	?	81

c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's $\frac{3}{8}$ rule.

d) Determine the order of convergence and the asymptotic error constant of the sequence generated by the recursive scheme: $x_{n+1} = \frac{x_n - 1 + x_n}{2}$

e) Write the algorithm of fixed point iteration method.

4. Write short answer on any two from the following questions: $4 \times 2 = 8$

a) Express $y = 2x^3 - 3x^2 + 3x - 10$ in a factorial notation and hence show that $\Delta^3 y = 12$.

b) Using Euler's method find an approximate value of y corresponding to $x = 1$, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$ and $y = 1$ when $x = 0$.

c) Apply Runge-Kutta fourth order method to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$.

d) Write the algorithm of Newton-Raphson method.

e) Solve the following system of equations by using Gauss Elimination method:

$$\begin{aligned} x - y + 2z &= 3 \\ 3x + 2y + 3z &= 5 \\ 3x - 4y - 5z &= -13 \end{aligned}$$

5. Answer any one from the following questions: $6 \times 1 = 6$

a) Derive Newton-Gregory interpolation formula for equal intervals.

b) Given the values:

x	5	6	9	11
$f(x)$	12	13	14	16

Evaluate $f(10)$ using Lagrange's interpolation formula.

c) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the following data:

x	0	1	2	3
$f(x)$	1	2	5	10

Hence interpolate at 0.5, 1.5, 2.5.

6. Answer any one from the following questions: $6 \times 1 = 6$

a) Find the first, second derivative of the function tabluated below at the point $x = 1.5$

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.00	13.625	24.00	38.875	59.00

b) Solve the different equation $\frac{dy}{dx} = -xy^2$, $y = 2$at $x = 0$ by modified Euler's method and obtain y at $x = 0.2$ in two steps of 0.1 each.

c) Derive General Quadrature formula for equal intervals and hence deduce the Trapezoidal rule from it.

7. Answer any one from the following questions: $6 \times 1 = 6$

a) Write the method for finding the solutions of the system of equations by using Gauss-Jordan method.

b) Solve the following system of equations by using LU decomposition method

$$\begin{aligned} x+5y+z &= 14 \\ 2x+y+3z &= 13 \\ 3x+y+4z &= 17 \end{aligned}$$

c) Find the root of the equation $x^4 - x - 10 = 0$ by using secant method.