DHANAMANJURI UNIVERSITY

Examination - 2024 (June)

Four-Year Course BA/B.Sc. 4^{th} Semester

Name of Programme	**:**	**B.A/B.Sc. Mathematics**
Paper Type	**:**	**Core-XI(Theory)**
Paper COde	**:**	**CMA-211**
Paper Title	**:**	**Mechanics**
Full Marks	**:**	**40**
Pass Marks	**:**	**16**
Duration	**:**	**2 Hours**

The figures in the margin indicate full marks for the questions
Answer the following questions:

1. Write the correct answers in each of the following: $1 \times 4 = 4$

a) A particle is executing a simple harmonic motion of amplitude a. The distance from the centre where it acquires a velocity one-third of the maximum velocity is

i) $\frac{2\sqrt{2}}{3}a$ ii) $\frac{3\sqrt{3}}{2}a$
iii) $\frac{\sqrt{2}}{3}a$ iv) $\frac{\sqrt{3}}{2}a$

b) At an apse point on a central orbit, a particle moves in a direction

i) Parallel to the radius vector
ii) Perpendicular to the tangential direction
iii) Perpendicular to the radius vector
iv) Bisecting the angle between the radial and cross-radial directions

c) A system of coplanar forces $F_1, F_2, \ldots, F_n$ acting on a body can be reduced to

i) Only to a single force of magnitude $F = \sum_{i=1}^{n} F_i$
ii) Only to a single couple whose moment is the algebraic sum of all moments of the forces about any point in the plane of the forces
iii) To a single force F together with a single couple of moment G
iv) None of the above

d) Four forces proportional to 1, 2, 3 and 4kgs are suspended from the corners of a square ABCD whose sides are of 3m. The distance of the centre of force from A is

i) $\frac{3}{5}\sqrt{37}/2$ iii) $\frac{5}{3}\sqrt{37}/2$

ii) $\frac{1}{3}\sqrt{37}/2$ iv) $\frac{1}{5}\sqrt{37}/2$

2. Answer very short answer on any three of the following: $1 \times 3 = 3$

a) The intrinsic equation of a curve is $S = 4a \sin \psi$. Find the radius of curvature at the point (S, ψ).

b) Write the general equation of motion when mass varies.

c) Can a uniform heavy rod rest in equilibrium within a smooth spherical bowl at any radius at an inclined position?

d) State the relation between angle of friction and the coefficient of friction.

3. Answer any one of the following: $3 \times 1 = 3$

a) A particle moves along the curve $r = 2a \cos \theta$ in such a manner that the acceleration towards the origin is always zero. Prove that the cross-radial acceleration varies as $\operatorname{cosec}^5 \theta$.

b) A particle describes a plane curve for which S and ψ vanish simultaneously with a uniform velocity v. If the acceleration at any point (S, ψ) be CV^2/C^2+S^2, C being a constant. Prove that $S = c \tan \psi$

c) A particle executes SHM between two points C and D. If the time period of oscillation is 2π, and if v is the velocity of the particle at any point P, show that $v^2 = CP \cdot DP$.

d) A heavy uniform rod of length $2a$ rests in equilibrium having one end against a smooth vertical wall and placed upon a peg at distance b from the wall. Show that the inclination of the rod to the horizontal is $\cos^{-1}\left(\frac{b}{a}\right)^{1/3}$.

e) State the three laws of statical friction.

f) Find the centroid of the area between the sine curve $y = \sin x$, $y = 0$, and the double ordinates $x = 0$ and $x = \pi$.

4. Answer any one of the following: $10 \times 1 = 10$

a) Prove that the acceleration $\vec{a}$ of a particle describing a plane curve is $\vec{a} = a_r \hat{e}_r + a_\theta \hat{e}_\theta$ where

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

and $\hat{e}_r$, $\hat{e}_\theta$ are unit vectors along radial and cross-radial directions. 10

b) A particle moves in a plane curve with a central acceleration F which is always directed towards a fixed point in the plane of the force. Prove that with usual notation $\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2}$, $u = \frac{1}{r}$. 10

c) A particle of mass m is projected vertically upwards under gravity, the resistance of air being proportional to mk times the velocity. Show that the greatest height attained by the particle is $V^2/g[\lambda - \log(1+\lambda)]$ where V is the terminal velocity and λV is the initial velocity of projection. 10

d) A particle of mass M is at rest and begins to move under the action of a constant force F in a fixed direction. It encounters the resistance of a stream of fine dust moving from opposite direction with velocity V which deposits matter on it at a constant rate ρ. Show that its mass will be m when it has travelled a distance $\kappa/\rho^2\,[m - M(1+\log m/M)]$ where $\kappa = F - \rho V$. 10

5. Answer any one of the following: $10 \times 1 = 10$

a) Prove that the necessary and sufficient condition that a system of coplanar forces acting on a rigid body may be in equilibrium are that the algebraic sums of the magnitudes of the resolved parts of the forces in any two mutually perpendicular directions should be separately zero and the algebraic sum of the moments of the forces about any point in their plane should also be zero. 10

b) Prove that the algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about that point. Discuss the different cases. 10

c) A uniform rod of weight W rests with the two ends in contact with two smooth planes inclined at angles α and β respectively to the horizon and intersecting in horizontal lines. If θ be the inclination of the rod to the vertical in one of the positions of equilibrium, then show that $\cot\theta = \frac{1}{2}[\cot\beta - \cot\alpha]$. Further, find the reactions at the ends of the rod. 10

d) A uniform heavy rod of weight W has its lower end attached to a hinge and its other end tied to an inextensible string which is fixed to a point vertically above the hinge. Draw a neat diagram to show the equilibrium position of the rod and show that the direction of action at the hinge will bisect the string. 10

6. Answer any one of the following: $10 \times 1 = 10$

a) A heavy body rests in limiting equilibrium on a rough inclined plane of inclination α to the horizon. If P be a force acting on the body at an angle θ to the plane, find the least P that just supports the body and when the body is at the point of sliding up the plane. 10

b) Prove that the C.G. of a uniform triangular lamina is at its centroid. Further, show that this C.G. is identical with that of any three equal particles placed at its vertices. 10

c) A uniform ladder rests in limiting equilibrium with the ends, one on a rough horizontal plane and the other against a smooth vertical wall. A man then ascends the ladder. Show that whatever be his weight, he cannot go more than halfway up the ladder. What happens if the horizontal plane also be smooth? Give reasons for your answer.

d) Find the C.G. of the arc which is in the first quadrant of the cycloid:

$$X = a(\theta + \sin\theta)$$
$$Y = a(1 - \cos\theta), \quad 0 \le \theta \le \pi.$$

10