DHANAMANJURI UNIVERSITY

Four-year course B.A/B.Sc 3rd Semester

DECEMBER-2022

Name of Programme	:	**B.A/B.Sc Mathematics (Honours)**
Paper Type	:	**Core-7(Theory)**
Paper Code	:	**CMA-207**
Paper Title	:	**Partial Differential Equations and Laplace Transform**
Full mark	:	**100**
Pass Mark	:	**40**
Duration	:	**3 Hours**

The figures in the margin indicate full marks for the question
Answer all the questions:

1. Choose and rewrite the correct answer for each of the following: **$1 \times 12 = 12$**

a) The partial differential equation formed by eliminating the arbitrary constants a and c from the equation $x^2+y^2+(z-c)^2=a^2$ is

i) $xp-yq=0$

ii) $xp+yq=0$

iii) $yp-xq=0$

iv) $yp+xq=0$

b) The complete integral of $pq=1$ is

i) $z=ax+\frac{by}{a}+c$

ii) $z=\frac{a}{x}+by+c$

iii) $z=x+\frac{y}{a}+b$

iv) $z = ax + \frac{y}{b} + c$

c) If $F(D, D')$ be homogeneous function of D and D' of degree n with constant coefficients, where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$ and $F(a, b) = 0$,then the particular integral of the equation$F(D, D')z = \phi(ax + by)$ is

i) $\frac{x^n}{a^n \cdot n!}\phi(ax + by)$

ii) $\frac{x^n}{b^n \cdot n!}\phi(ax + by)$

iii) $\frac{y^n}{a^n \cdot n!}\phi(ax + by)$

iv) $\frac{y^n}{b^n \cdot n!}\phi(ax + by)$

d) The Laplace Transform of te^{at} is

i) $\frac{1}{s - a}$

ii) $\frac{1}{s + a}$

iii) $\frac{1}{(s - a)^2}$

iv) $\frac{1}{(s + a)^2}$

2. Write very answer for each of the following: $1 \times 12 = 12$

a) What is a linear partial differential equation?

b) What is the geometrical interpretation of Lagrange's first order linear partial differential equation?

c) If Jacobian of two functions $\xi = \xi(x, y)$ and $\eta = \eta(x, y)$ with respect to two variables x and y is zero, what cay you say about the two functions ξ and η.

d) What is the geometrical interpretation of singular integral of a non-linear partial differential equation of first order?

e) What are compatible systems of first order partial differential equations?

f) Write the complete integral of the equation of the form $f(p, q) = 0$?

g) Write the general form of a second order linear partial differential equation in two independent variables?

h) Determine the region in which the equation $\left(x^2-1\right)u_{xx}+2yu_{xy}-u_{yy}=0$ is hyperbolic.

i) Write λ-quadratic of the Monge's equation $Rr+Ss+Tt+U\left(rt-s^2\right)=V$,where the symbols have their usual meanings.

j) Define Laplace Transform of a function $f(t)$

k) Define function of class A.

l) Find the inverse Laplace Transform of $\dfrac{s}{s^2-a^2}$

3. Choose any twelve and rewrite short answers: 3 × 12 = 36

a) Find the partial differential equation by eliminating the arbitrary constants a and b from the equation $(x-a)^2+(y-b)^2+z^2=r^2$.

b) Form a partial differential equation by eliminating the arbitrary function F from the equation $F(u,v)=0$,where u and v are known functions of x, y and z.

c) Find the general solution of the equation:
$x\left(y^2+z\right)p-y\left(x^2+z\right)q=z\left(x^2-y^2\right)$

d) How do you obtain of a first order non-linear partial differential equation? What is the geometrical interpretation of a general integral?

e) Find the complete integral of the equation: $z^2\left(p^2z^2+q^2\right)=1$

f) Find the complete integral of the equation: $\dfrac{p^2}{x}-\dfrac{q^2}{y}=\dfrac{1}{z}\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$

g) Find the complete integral of the Clairaut Equation by using Charpit's Method. What does the complete integral of a Clairaut Equation represent geometrically?

h) show that: $\begin{bmatrix} A^{\bullet} & \frac{B^{\bullet}}{2} \\ \frac{B^{\bullet}}{2} & c^{\bullet} \end{bmatrix} = \begin{bmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{bmatrix} \begin{bmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{bmatrix} \begin{bmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{bmatrix}^{\gamma}$ where
$A^{\bullet}=A\xi_x^2+B\xi_x\xi_y+C\xi_y^2$, $B^{\bullet}=2A\xi_x\eta_x+B\left(\xi_x\eta_y+\xi_y\eta_x\right)+2C\xi_y\eta_y$ and
$C^{\bullet}=A\eta_x^2+B\eta_x\eta_y+C\eta_y^2$

i) If $F(D,D') = D - mD'$, find the particular integral of the equation $F(D,D')z = f(x,y)$ using Lagrange's Method, where $D = \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$

j) Find the general solution of the equation: $r + 5s + 6t = (y-2x)^{-1}$

k) Find the general solution of the equation: $\left(2DD' + D'^2 - 3D'\right) z = 5\cos(3x-2y)$

l) Show that $f(x) = x^n$ is of exponential order as $x \to \infty$,n being any positive integer.

m) If $L\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a) & , t > a \\ 0 & , t < a \end{cases}$,prove that

$L\{G(t)\} = e^{-as} f(s)$

n) Show that $\int_0^{\infty} te^{-3t} \sin t \, dt = \frac{3}{50}$.

o) Evaluate: $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$

4. Answer any two of the following: $2 \times 6 = 12$

a) Solve: $y^2(x-y)p + x^2(y-x)q = z\left(x^2+y^2\right)$

b) Find the general integral of the partial differential equation $(2xy-1)p + \left(z-2x^2\right)q = 2(x-yz)$ and also the particular integral which passes through the line $x = 1$, $y = 0$.

c) Reduce the equation $u_x - u_y = u$ to canonical form and obtain the general solution.

d) Solve the equation $y^2u_x^2 + x^2u_y^2 = (xyu)^2$, $u(x,0) = 3e^{\frac{x^2}{4}}$ by the method of separation of variables of the form

$u(x,y) = f(x).g(y)$

5. Answer any two of the following questions: $6 \times 2 = 12$

a) Prove that the necessary and sufficient condition for the first order PDEs: $f(x,y,z,p,q) = 0$ and $g(x,y,z,p,q) = 0$ to be compatible is that the Jacobi bracket $[f,g] \equiv \frac{\partial(f,g)}{\partial(x,p)} + p\frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\frac{\partial(f,g)}{\partial(z,q)} = 0$

b) Describe Charpit's method for solving the first order non-linear PDE.

c) Apply Charpit's method to solve the equation: $2z + p^2 + qy + 2y^2 = 0$

d) Apply Jacobi's method to find the complete integral of: $P_1P_2P_3 = Z^3x_1x_2x_3$

6. Answer any two of the following questions: $6 \times 2 = 12$

a) Reduce to canonical form and find the general solution of the equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$.

b) Solve: $r + s - 6t = y\cos x$

c) Solve: $x^2\frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x \partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3y^4$

d) Apply Monge's method to find the general integral of $y^2r - 2ys + t = p + 6y$

7. Answer any two of the following questions: $6 \times 2 = 12$

a) Prove that if a function F(t) is piece-wise continuous in every finite interval in the range $t \geq 0$ and is of exponential order a as $t \to \infty$, then Laplace transform of F(t) exists for all $s > a$.

b) If $L\{F(t)\} = f(s)$, prove that $L\{t^nF(t)\} = (-1)^n\frac{d^n}{ds^n}f(s)$, $n = 1,2,3\ldots$

c) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$

d) Apply Laplace transform to solve $\frac{d^2y}{dt^2} + y = 6\cos 2t$ given that $y = 3$, $\frac{dy}{dt} = 1$ where $t = 0$.

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