CMA – 207

2021

(December)

MATHEMATICS

(Honours)

SEVENTH PAPER

(Partial Differential Equations and Laplace Transform)

Full Marks : 100
Pass Marks : 40

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **all** questions

1. Choose and rewrite the correct answer : 1x5=5

(a) The partial differential equation of all surfaces of revolution having z-axis as the axis of rotation is

(i) $xp - yq = 0$

(ii) $xp + yq = 0$

(iii) $yp - xq = 0$

(iv) $yp + xq = 0$

(b) If $F(D, D')$ be homogeneous function of D and D' of degree n with constant coefficients, where $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$, then the value of $\frac{1}{F(D, D')}\phi^n(ax+by)$ is

(i) $\frac{1}{F(a,b)}\phi^n(ax+by)$

(ii) $\frac{a^n}{F(a,b)}\phi(ax+by)$

(iii) $\frac{b^n}{F(a,b)}\phi(ax+by)$

(iv) $\frac{1}{F(a,b)}\phi(ax+by)$

(c) The complete integral of $p = e^q$ is

(i) $z = ay + x\log a + c$

(ii) $z = \frac{a}{x} + y\log a + c$

(iii) $z = ax + y\log a + c$

(iv) $z = a + xy\log a + c$

(d) The complete integral of the equation $z = px - qy + 3p - 2q$ represents a family of planes through a fixed point in space. Then, the fixed point is

(i) (3,2,0)

(ii) (-3,2,0)

(iii) (3,-2,0)

(iv) (-3,-2,0)

(e) The inverse Laplace transform of $f\left(s-a\right)$ is

(i) $e^{-at}L^{-1}\{f(s)\}$

(ii) $e^{-at}L\{f(s)\}$

(iii) $e^{at}L^{-1}\{f(s)\}$

(iv) $e^{at}L\{f(s)\}$

2. Write very short answer for each of the following : 1x5=5

(a) Give the geometrical interpretation of Lagrange's linear partial differential equation of first-order $Pp + Qq = R$.

(b) Write the general form of a linear homogeneous partial differential equation with constant coefficients.

(c) Find the particular integral of $4r - 4s + t = 16\ \log(x + 2y)$.

(d) What is the general solution of the partial differential equation $zp = -x$.

(e) If $L^{-1}\{f(s)\} = F(t)$ and $F(0) = 0$, then prove that $L^{-1}\{sf(s)\} = F'(t)$.

3. Write short answer for each of the following : 3x10=30

(a) Obtain a partial differential equation by eliminating the arbitrary function f from the equation $f\left(x + y + z, xyz\right) = 0$.

(b) Find $L\left\{\int_0^t \frac{\sin x}{x}\,dx\right\}$.

(c) Solve the equation $(D^4 - 2D^3D' + 2DD'^3 - D'^4)z = 0$.

(d) Obtain the general solution of the equation $xu_x + yu_y = nu$.

(e) Solve: $r + 5s + 6t = (y - 2x)^{-1}$

(f) Find the singular integral of $z = px + qy + \sqrt{\alpha p^2 + \beta q^2 + \gamma}$.

(g) Find the general solution of the equation $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.

(h) Solve: $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$

(i) Evaluate: $L^{-1}\left\{\frac{e^{-5s}}{(s-2)^4}\right\}$

(j) Solve: $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$

4. Answer any *two* parts : 6x2=12

(a) Find the solution of the equation $u(x+y)u_x + u(x-y)u_y = x^2 + y^2$ with the Cauchy data $u = 0$ on $y = 2x$.

(b) Reduce the equation $u_x + u_y = u$ to canonical form and obtain the general solution.

(c) Apply the method of separation of variables $u(x, y) = f(x)\cdot g(y)$ to solve the equation $u_x + u = u_y$, $u(x,0) = 4e^{-3x}$.

5. Answer any *two* parts : 6x2=12

(a) Solve: $z(x + y)p + z(x - y)q = x^2 + y^2$

(b) Solve: $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

(c) Find the equation of the integral surface of the partial differential equation $2y(z-3)p + (2x - z)q = y(2x-3)$ which pass through the circle $z = 0$, $x^2 + y^2 = 2x$.

6. Answer any *two* parts : 6x2=12

(a) Solve $(x^2 + y^2)(p^2 + q^2) = 1$ by reducing it to one of the standard forms.

(b) Apply Charpit's method to solve the equation $(p + q)(px + qy) = 1$.

(c) Solve by Jacobi's method: $2p_1x_1x_3 + 3p_2x_3^2 + p_2^2p_3 = 0$.

7. Answer any *two* parts : 6x2=12

(a) Reduce to canonical form and find the general solution of the equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} + xyu_x + y^2u_y = 0$.

(b) Solve: $x^2r - y^2t = xy$.

(c) Apply Monge's method to find the general integral of $r - t\cos^2 x + p\tan x = 0$.

8. Answer any *two* parts : 6x2=12

(a) Apply convolution theorem to evaluate $L^{-1}\left\{\dfrac{1}{s^2(s+1)^2}\right\}$

(b) Solve by using Laplace transform $\dfrac{d^2y}{dt^2} + y = t\cos 2t,\quad t > 0,$

given that $y = 0 = \dfrac{dy}{dt}$ when $t = 0$.

(c) Apply Laplace transform to solve $\dfrac{\partial y}{\partial x} - \dfrac{\partial y}{\partial t} = 1 - e^{-t},\ 0 < x < 1,\ t > 0$,

given that $y(x,0) = x$.

***