

DHANAMANJURI UNIVERSITY

Examination - 2024 (June)

Four-Year Course BA/B.Sc. 4th Semester

Name of Programme : B.A/B.Sc. Mathematics
Paper Type : Core-VI (Theory)
Paper Code : CMA-106
Paper Title : Computer Algebra System
Full Marks : 40
Pass Marks : 16
Duration : 2 Hours

The figures in the margin indicate full marks for the questions

Answer the following questions:

1. Choose and rewrite the correct answer:

$1 \times 3 = 3$

a) If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\text{div } \vec{a}$ is:

- i) 2,
- ii) 3,
- iii) -3,
- iv) -2.

b) The equation of the cylinder generated by the lines parallel to the z-axis and passing through the curve of intersection of the plane $lx + my + nz = p$ and the surface $ax^2 + by^2 + cz^2 = 1$ is:

- i) $ax^2 + by^2 + c\left(\frac{p - lx - my}{n}\right)^2 = 1$,
- ii) $cz^2 + a\left(\frac{p - nz}{l}\right)^2 = 1$,
- iii) $cz^2 + by^2 + a\left(\frac{p - nz - my}{l}\right)^2 = 1$,
- iv) $cz^2 + b\left(\frac{p - nz}{l}\right)^2 = 1$.

c) The equation of hyperbolic paraboloid is:

i) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

ii) $\frac{x^2}{a^2} = \frac{2z}{c} - \frac{y^2}{b^2}$,

iii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c}$,

iv) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$.

2. Write very short answers for each of the following:

$1 \times 5 = 5$

a) Define reciprocal vectors.

b) Give an example of a vector point function.

c) Write the formula to find the radius of a sphere whose equation is given by $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

d) Define a right circular cone.

e) What do you mean by hyperboloid of one sheet?

3. Write short answers (any two) of the following:

$3 \times 2 = 6$

a) The acceleration of a particle at time t is given by

$$\vec{a} = 18 \cos t \hat{i} - 8 \sin 2t \hat{j} + 6t \hat{k}.$$

If the velocity \vec{v} be zero at time $t = 0$, find \vec{v} at any time t .

b) Find the equation of the sphere on which the circle given by $x + y + z + 3 = 0$ and $x^2 + y^2 + z^2 = 9$ is a great circle.

c) The axis of a right circular cylinder is $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$ and its radius is 5. Find its equation.

4. Answer any two of the following:

$4 \times 2 = 8$

a) Using Green's theorem, evaluate $\int_C (x^2 y dy + x^2 dx)$, where C is the boundary described counterclockwise of the triangle with vertices $(0, 0), (1, 0), (1, 1)$.

b) Find the equation of the curve in which the plane $z = h$ cuts the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and find the area enclosed by the curve.

c) Prove that the point (1,2,3) and (1,2,1) are conjugate with respect to the conicoid $2x^2 + 3y^2 - 4z^2 = 2$.

5. Answer any one of the following: **6 × 1 = 6**

a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $F(x, y, z) = -y^2\hat{i} + x\hat{j} + z^2\hat{k}$ and C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$.

b) Apply Stokes' theorem to find the value of $\int_C (y dx + z dy + x dz)$, where C is the curve of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

6. Answer any one of the following: **6 × 1 = 6**

a) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 14$ at the point $(1, -2, 3)$.

b) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular generators, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

7. Answer any one of the following: **6 × 1 = 6**

a) Find the equation of the tangent plane to the paraboloid $\frac{x^2}{5} - \frac{y^2}{3} = 2z$ parallel to the plane $2x - 3y + z = 0$.

b) Prove that the plane $x + y + z = 1$ touches the conicoid $\frac{x^2}{3} + \frac{y^2}{2} - \frac{z^2}{4} = 1$.