

# DHANAMANJURI UNIVERSITY

Examination - 2024 (June)

Four-Year Course BA/B.Sc. 4<sup>th</sup> Semester

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|-------------------|---------------------------|
| Name of Programme | : B.A/B.Sc. Mathematics   |
| Paper Type        | : Core-VI (Theory)        |
| Paper Code        | : CMA-106                 |
| Paper Title       | : Computer Algebra System |
| Full Marks        | : 40                      |
| Pass Marks        | : 16                      |
| Duration          | : 2 Hours                 |

*The figures in the margin indicate full marks for the questions  
Answer the following questions:*

**1. Choose and rewrite the correct answer:**

**1 × 3 = 3**

a) If  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ , then  $\text{div } \vec{a}$  is:

- i) 2,
- ii) 3,
- iii) -3,
- iv) -2.

b) The equation of the cylinder generated by the lines parallel to the z-axis and passing through the curve of intersection of the plane  $lx + my + nz = p$  and the surface  $ax^2 + by^2 + cz^2 = 1$  is:

- i)  $ax^2 + by^2 + c \left( \frac{p - lx - my}{n} \right)^2 = 1,$
- ii)  $cz^2 + a \left( \frac{p - nz}{l} \right)^2 = 1,$
- iii)  $cz^2 + by^2 + a \left( \frac{p - nz - my}{l} \right)^2 = 1,$
- iv)  $cz^2 + b \left( \frac{p - nz}{l} \right)^2 = 1.$

c) The equation of hyperbolic paraboloid is:

i)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$

ii)  $\frac{x^2}{a^2} = \frac{2z}{c} - \frac{y^2}{b^2},$

iii)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c},$

iv)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}.$

**2. Write very short answers for each of the following:**

**1 × 5 = 5**

- a) Define reciprocal vectors.
- b) Give an example of a vector point function.
- c) Write the formula to find the radius of a sphere whose equation is given by  $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ .
- d) Define a right circular cone.
- e) What do you mean by hyperboloid of one sheet?

**3. Write short answers (any two) of the following:**

**3 × 2 = 6**

- a) The acceleration of a particle at time  $t$  is given by  
 $\vec{a} = 18 \cos t \hat{i} - 8 \sin 2t \hat{j} + 6t \hat{k}.$   
If the velocity  $\vec{v}$  be zero at time  $t = 0$ , find  $\vec{v}$  at any time  $t$ .
- b) Find the equation of the sphere on which the circle given by  $x + y + z + 3 = 0$  and  $x^2 + y^2 + z^2 = 9$  is a great circle.
- c) The axis of a right circular cylinder is  $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$  and its radius is 5. Find its equation.

**4. Answer any two of the following:**

**4 × 2 = 8**

- a) Using Green's theorem, evaluate  $\int_C (x^2 y dy + x^2 dx)$ , where  $C$  is the boundary described counterclockwise of the triangle with vertices  $(0,0), (1,0), (1,1)$ .

- b) Find the equation of the curve in which the plane  $z = h$  cuts the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  and find the area enclosed by the curve.
- c) Prove that the point  $(1,2,3)$  and  $(1,2,1)$  are conjugate with respect to the conicoid  $2x^2 + 3y^2 - 4z^2 = 2$ .

**5. Answer any one of the following:**

**6 × 1 = 6**

- a) Evaluate  $\int_C \vec{F} \cdot d\vec{r}$ , where  $F(x,y,z) = -y^2\hat{i} + x\hat{j} + z^2\hat{k}$  and  $C$  is the curve of intersection of the plane  $y + z = 2$  and the cylinder  $x^2 + y^2 = 1$ .
- b) Apply Stokes' theorem to find the value of  $\int_C (ydx + zdy + xdz)$ , where  $C$  is the curve of  $x^2 + y^2 + z^2 = a^2$  and  $x + z = a$ .

**6. Answer any one of the following:**

**6 × 1 = 6**

- a) Find the equation of the tangent plane to the sphere  $x^2 + y^2 + z^2 = 14$  at the point  $(1, -2, 3)$ .
- b) Prove that the plane  $ax + by + cz = 0$  cuts the cone  $yz + zx + xy = 0$  in perpendicular generators, if  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$ .

**7. Answer any one of the following:**

**6 × 1 = 6**

- a) Find the equation of the tangent plane to the paraboloid  $\frac{x^2}{5} - \frac{y^2}{3} = 2z$  parallel to the plane  $2x - 3y + z = 0$ .
- b) Prove that the plane  $x + y + z = 1$  touches the conicoid  $\frac{x^2}{3} + \frac{y^2}{2} - \frac{z^2}{4} = 1$ .