DHANAMANJURI UNIVERSITY

Examination- 2024 (June)

Four-year course B.A/B.Sc. 2nd Semester

Name of Programme : B.A/B.SC Mathematics
Paper Type : Core-V (Theory)
Paper Code : CMA-105
Paper Title : Differential equations
Full Marks : 40
Pass Marks : 16

Duration: 2 Hours

The figures in the margin indicate full marks for the questions
Answers the following questions:

1. **Choose and rewrite the correct answer for the following questions:** $1 \times 3 = 3$

a) The sum of the order and degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is

i) 2 ii) 3
iii) 4 iv) 5

b) The complete solution of the differential equation $p^2 + p = 6$ (where $p = \frac{dy}{dx}$) is

i) $(y - 3x - c)(y - 2x - c) = 0$
ii) $(y + 3x - c)(y + 2x - c) = 0$
iii) $(y + 3x - c)(y - 2x - c) = 0$
iv) $(y - 3x - c)(y + 2x - c) = 0$

c) The particular integral of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$ is

i) $\frac{1}{9}e^{5x}$ ii) $\frac{1}{10}e^{5x}$

iii) $\frac{1}{11}e^{5x}$ iv) $\frac{1}{12}e^{5x}$

2. Write very short answers on any five of the following: $1 \times 5 = 5$

a) Write the solution of the differential equation $ydx - xdy = 0$.

b) When is the equation $Pdx + Qdy + Rdz = 0$ said to be exact?

c) Define Clairaut's differential equation.

d) Write the word equation for drug assimilation into the blood.

e) Find the complementary function of $(D^2 + D - 6)y = x$.

f) Define the Wronskian.

3. Answer any two of the following questions: $3 \times 2 = 6$

a) Solve: $\frac{dy}{dx} + y\tan x = \sec x$.

b) Solve: $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$ Interprete your result geometrically.

c) Solve: $y = 2px - xp^2$

d) Solve the differential equation of exponential decay and radioactivity $\frac{dN}{dt} = -kN$, (where k is a positive of proportionality) with the initial condition $N(t_o) = n_o$.

e) If $y_1(x) = \sin 3x$ *and* $y_2(x) = \cos 3x$ are the two solutions of $y'' + 9y = 0$, show that $y_1(x)$ *and* $y_2(x)$ are linearly independent solutions.

4. Answer any two of the following questions: $4 \times 2 = 8$

a) Define Bernoulli's equation. Show that such an equation can be reduced to the form of linear differential equation.

b) Show that the differential equation $y \sin 2x\, dx - (y^2 + \cos^2 x)dy = 0$ and hence solve it and given that $y = 1$ *where* $x = 0$.

c) Solve the differential equation $p^2 + 2py \cot x = y^2$.

d) Solve the differential equation $\sin^2 x \frac{d^2y}{dx^2} = 2y$ given that $y = \cot x$ is a solution.

e) Solve : $\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right)y = xe^x$.

5. Answer any one questions from the following: $6 \times 1 = 6$

a) Define exact differential equation. Examine whether the differential equation $x\, dx + y\, dy = \frac{a^2(xdy - ydx)}{x^2+y^2}$ is exact or not. If it be exact, then find its solution. Also find the particular solution by giving $y = 1$ *where* $x = 0$.

b) Find the orthogonal trajectories of

$$\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1, \textit{where } \lambda \textit{ is arbitrary}$$

c) Define total differential equation. Show that the total differential equation $Pdx + Qdy + Rdz = 0$ will be integrable if $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial p}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$ State the names of method that can be used in solving the total differential equations.

6. Answer any one questions from the following: $6 \times 1 = 6$

a) Define singular solution. Find the complete primitive and singular solution of $y = px + \sqrt{b^2 + a^2p^2}$ Interprete your result geometrically.

b) The acceleration of a moving particle being proportional to the cube of the velocity and negative, find the distance passed over in time t, the initial velocity being v_0 and the distance being measured from the position of the particle at the time $t = 0$.

c) A radio active substance has a half-life of h days. Find a formula its mass m in terms of the time t, if the initial mass is m_o. What is its initial decay rate?

7. Answer any one questions from the following: $6 \times 1 = 6$

a) Solve: $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$

b) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = x$

c) Solve by the method of undetermined co-efficients, the differential equation $\frac{d^2y}{dx^2} + 4y = \sin 2x.$

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