Dhanamanjuri University

JUNE 2022

Name of Programme	**:**	**B.Sc Mathematics (Honours)**
Semester	**:**	**II**
Paper Type	**:**	**Core IV**
Paper Code	**:**	**CMA-104**
Paper Title	**:**	**Differential Equations**
Full Marks	**:**	**100**
Duration	**:**	**3 Hours**

1. Choose and rewrite the correct answer for each of the following questions:

(a) The integrating factor of the group of terms as being part of an exact differential equation $\frac{xdy - ydx}{x^2}$, is

(i) $d\left(\frac{x}{y}\right)$
(ii) $d\left(\log \frac{y}{x}\right)$
(iii) $d\left(\log \frac{x}{y}\right)$
(iv) $d\left(\frac{y}{x}\right)$

(b) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} + 5\left(\frac{dy}{dx}\right)^4 + 8y = \log x$ is

(i) (3,4)
(ii) (2,3)
(iii) (4,3)
(iv) (3,1)

(c) The complementary solution of the equation $(D^2 + 4)y = 0$ is

(i) $\cos 2x + i \sin 2x$
(ii) $e^{-2x}(\cos x + \sin x)$
(iii) $\cos 2x + \sin 2x$
(iv) $e^{2x}(\cos x + \sin x)$

(d) In the Lake Pollution Model of equation $\frac{dM}{dt} = C_{in}(t) \otimes Q_{in}(t) - C_{out}(t) \otimes Q_{out}(t)$, the volumetric flow rate through the lake is denoted by:

(i) C
(ii) M
(iii) t
(iv) Q

(e) The solution of the differential equation $\tan y \, dx + \tan x \, dy = 0$ is

(i) $\sin x \sin y = c$
(ii) $\sin x \cos y = c$
(iii) $\cos x \cos y = c$
(iv) $\cos x \sin y = c$

2. Write very short answer for each of the following questions:

(a) What is an integrating factor?

(b) Define Wronskian of a differential equation.

(c) Write the length formula of the Cartesian tangent.

(d) Define a non-linear differential equation.

(e) What do you mean by orthogonal trajectory?

(f) Name the two solutions involved in the general solution of a non-homogeneous differential equation.

(g) When the method of variation of parameters be applied for solving a non-homogeneous linear differential equation?

(h) What do you mean by battle model in a differential equation?

(i) Why is drug assimilation into the blood modeled?

(j) What is that condition that the equation $Mdx + Ndy = 0$ will be exact?

(k) Which differential equation is known as the extension form of Clairaut's equation?

3. Write short answers for each of the following questions:

(a) What is Mathematical Modelling? Write the applications of Mathematical Modelling.

(b) If the complementary function (y_c) of a differential equation is $c_1e^x + c_2e^{-x} + c_3e^{2x} + c_4e^{-2x}$, then find the roots of the auxiliary equation (AE).

(c) Solve $(x^4e^x - 2axy^2)dx + 2ax^2ydy = 0$

(d) Define homogeneous linear equation or Cauchy-Euler equations.

(e) What is the condition that the equation of the type $Mdx + Ndy = 0$ are function of x and y to be exact?

(f) What are equilibrium points in differential equation?

(g) Define singular solution. What is the singular solution of the differential equation of the form $y = px + \frac{a}{p}, wherep = \frac{dy}{dx}$

(h) Solve $x^2\frac{dy}{dx} + y = 1$

4. Answer any two parts:

(a) Solve the equation $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$

(b) Solve the Clairaut's equation $p(p^2 + xy) = p^2(x + y)$ and obtain the singular solution

(c) In a certain culture of bacteria, the rate of increase is proportional to the number present. If it be found that their number double in 4 hours. What will be their number at the end of 12 hours?

(d) Solve $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$

(e) Prove that $e^{\int pdx}$ is an integrating factor to the linear equation $\frac{dy}{dx} + Py = Q$ where P and Q are function of x alone or constant

(f) Define the term Epidemiology. Derive a mathematical model to solve an epidemic problem by assuming N_i = the number of infected student at any time and N_u = the number of uninfected student. Also draw the logistic curve of the above model.

(g) What is the Bernoulli's form of ordinary differential equation? Show that such an equation can be reduced to the linear form of a differential equation.

(h) Write down the method of solution of the equation of the form

$$y = px + f(p) \quad \text{where } p = \frac{dy}{dx}$$

5. Answer any two parts:

(a) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x + e^{3x}$

(b) Write the necessary condition for the integrability of a Pfaffian differential equation $Pdx + Qdy + Rdz = 0$

(c) Show that the orthogonal trajectories of the system of coaxial circles $x^2+y^2+2\lambda x+c = 0$ form another system of coaxial circles $x^2 + y^2 + 2\mu y - c = 0,$ where λ and μ are parameters and c is a given constant.

6. Answer any two parts:

(a) Solve, by the method of variation of parameters, the equation $\frac{d^2y}{dx^2} + y = \sec x \tan x$

(b) Solve $yz \log z\, dx - zx \log z\, dy + xyz\, dz = 0$

(c) The rate at which radioactive substance is proportional to the number of atoms present at any instant. If initially there are N_0 atoms at times t_0, find the number of atoms N at any instant t.

7. Answer any two parts:

(a) Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(b) Reduce the equation $\sin y\frac{dy}{dx} = \cos x(2\cos y - \sin^2 x)$ to a linear differential equation and solve it.

(c) Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}\cos 2x$