

DHANAMANJURI UNIVERSITY

Examination, 2023 (Dec)

Four year course B.Sc. 1st Semester

Name of Programme : B.A/B.Sc. Mathematics (Honours)

Semester : I

Paper Type : Core-III (Theory)

Paper Code : CMA-103

Paper Title : Analytic Geometry

Full Marks : 80

Duration: 3 Hours

The figures in the margin indicate full marks for the questions

All the questions.

d) The condition that the line $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$ is

i) $c = \frac{a}{m}$

ii) $c = \frac{1}{m}$

iii) $c = am$

iv) $c = \frac{1}{a}$

2. Write very short answer for each of the following questions:

1 × 10 = 10

a) Find the equation of the straight line $\frac{x}{a} + \frac{y}{b} = 2$ when the origin is transformed to the point (a, b) without changing direction of axes.

b) Write down the formulae of transformation from one pair of rectangular axes to another with same origin.

c) What is value of $\tan \theta$, if θ is the angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$.

d) Write the condition that a second degree general equation represent a pair of intersecting lines.

e) Transform the equation $5x + 3y = 3$ to parallel axes through the new origin $(2, -1)$.

f) Find the asymptotes of the hyperbola $xy + 4x + 3y + 5 = 0$.

g) Find the center of the conic given by the equation $3x^2 - 8xy + 7y^2 - 4x + 2y - 7 = 0$.

h) Under what condition of the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a circle

i) Write the equation of the chord of the conic $\frac{l}{r} = 1 + e \cos \theta$ joining the two points on the conic, whose vectorial angles are $(\alpha + \beta)$ and $(\alpha - \beta)$.

j) write the equation of conics which are confocal with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

3. Write short answer for each of the following questions:

$3 \times 10 = 30$

- a) Find the transform equation of the curve $(x + 2y + 5)(2x - y + 5) = 5$ when the two perpendicular lines $(x + 2y + 5) = 0$ and $(2x - y + 5 = 0)$ are taken as coordinate axes.
- b) Prove that a homogeneous equation of second degree $ax^2 + 2hxy + by^2 = 0$. represent a pair of straight lines through the origin.
- c) Prove that the equation $2x^2 - 7xy + 3y^2 + x + 7y - 6 = 0$ represents a pair of straight lines inclined to each other at 45° .
- d) Find the equation of the chord of contact of tangent drawn to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the external point (x_1, y_1)
- e) Obtain the equation of the circle which cuts orthogonally each of the 3 circles

$$x^2 + y^2 = 16, \quad x^2 + y^2 - 14x + 40 = 0 \text{ and}$$

$$x^2 + y^2 - 12y + 32 = 0.$$
- f) Find the asymptote of the hyperbola

$$2x^2 - 5xy - 3y^2 - 5x - 3y - 21 = 0.$$
- g) Find the condition that the line $lx + my + n = 0$ may represent a tangent line to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$$
- h) Find the condition that the normals at the points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ may be concurrent.
- i) Develop the polar equation of a conic with a focus as the pole and line joining the focus to the corresponding vertex as the initial line.
- j) Prove that the semi-latus rectum of any conic (in polar) is the harmonic mean between the segments of any focal chord.

4. Write the answer of the following questions:

a) Let (x, y) and (x', y') be the coordinate of a point referred to the original axes and new axes obtained by rotating an angle θ in the anti-clockwise direction. Develop the formulae to change (x, y) to (x', y') and vice versa.

Or

Define invariants. If by a rotational transformation about an origin the expression $ax^2 + 2hxy + by^2$ transforms to $a'x'^2 + 2h'x'y' + b'y'^2$ prove that $a + b = a' + b'$ and $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$.

b) Show that the equation $ab(x^2 + y^2) + (a^2 + b^2)xy + aby^2 + ab(a - b)(x - y) - a^2b^2 = 0$ represents two straight lines equidistant from the origin.

Or

Prove that the equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ represents a pair of parallel straight lines and find the distance between them.

5. Answer any two questions from the following:

a) Show that the standard form of the conic $9x^2 + 24xy + 16y^2 - 126x + 82y - 59 = 0$ represents the equation of a parabola whose axis is the new x' - axis and whose latus rectum is of length 6.

b) Prove that the general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents ellipse or hyperbola according to $ab - h^2$ as negative or positive.

c) Define pole and polar. Find the pole of the straight line $lx + my + n = 0$ with respect to the circle $x^2 + y^2 = a^2$.

6. Answer the following questions:

a) Define centre of a conic. Develop the formulae of the centre of the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

Or

Differentiate diameter and conjugate diameter of a conic. Find the condition that the pair of lines $Ax^2 + 2Hxy + By^2 = 0$ may be conjugate diameter of the conic $ax^2 + 2hxy + by^2 = 1$.

b) If the straight line $r \cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e \cos\theta$. Prove that $(l \cos \alpha - ep)^2 + l^2 \sin^2 \alpha = p^2$.

Or

Prove that the shortest focal chord of conic is latus rectum.
