

DHANAMANJURI UNIVERSITY

Examination, 2023 (Dec)

Four year course B.Sc. 1st Semester

Name of Programme : B.A/B.Sc. Mathematics (Honours)

Semester : I

Paper Type : Core-III (Theory)

Paper Code : CMA-103

Paper Title : Analytic Geometry

Full Marks : 80

Pass Marks : 32

Duration: 3 Hours

*The figures in the margin indicate full marks for the questions
All the questions.*

1. Choose and rewrite the correct answer for each of the following questions:

1 × 4 = 4

a) If by rotation of the rectangular axes, the equation

$$7x^2 + 18xy - 7y^2 = 1 \text{ reduces to the form } ax^2 + by^2 = 1$$

then the value of a is

i) -10

ii) 10

iii) 20

iv) -20

b) The equation of the bisectors of the angles between the pair of lines $ax^2 + 2hxy + by^2 = 0$ is

i) $\frac{x^2 - y^2}{a - b} = \frac{xy}{ab}$

ii) $\frac{x^2 + y^2}{a + b} = \frac{xy}{ab}$

iii) $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

iv) $\frac{x^2 + y^2}{a + b} = \frac{xy}{h}$

c) The polar (x', y') of the line $2x + 3y = 3$ with respect to the circle $x^2 + y^2 = 16$ is

i) $x' = 4, y' = 6$

ii) $x' = 6, y' = 4$

iii) $x' = -4, y' = 6$

iv) $x' = 4, y' = -6$

- d) The condition that the line $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$ is

i) $c = \frac{a}{m}$

ii) $c = \frac{1}{m}$

iii) $c = am$

iv) $c = \frac{1}{a}$

2. Write very short answer for each of the following questions:

1 × 10 = 10

- a) Find the equation of the straight line $\frac{x}{a} + \frac{y}{b} = 2$ when the origin is transformed to the point (a, b) without changing direction of axes.
- b) Write down the formulae of transformation from one pair of rectangular axes to another with same origin.
- c) What is value of $\tan \theta$, if θ is the angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$.
- d) Write the condition that a second degree general equation represent a pair of intersecting lines.
- e) Transform the equation $5x + 3y = 3$ to parallel axes through the new origin $(2, -1)$.
- f) Find the asymptotes of the hyperbola $xy + 4x + 3y + 5 = 0$.
- g) Find the center of the conic given by the equation $3x^2 - 8xy + 7y^2 - 4x + 2y - 7 = 0$.
- h) Under what condition of the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a circle
- i) Write the equation of the chord of the conic $\frac{l}{r} = 1 + e \cos \theta$ joining the two points on the conic, whose vectorial angles are $(\alpha + \beta)$ and $(\alpha - \beta)$.
- j) write the equation of conics which are confocal with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

3. Write short answer for each of the following questions:

$3 \times 10 = 30$

- a) Find the transform equation of the curve $(x + 2y + 5)(2x - y + 5) = 5$ when the two perpendicular lines $(x + 2y + 5) = 0$ and $(2x - y + 5) = 0$ are taken as coordinate axes.
- b) Prove that a homogeneous equation of second degree $ax^2 + 2hxy + by^2 = 0$ represent a pair of straight lines through the origin.
- c) Prove that the equation $2x^2 - 7xy + 3y^2 + x + 7y - 6 = 0$ represents a pair of straight lines inclined to each other at 45° .
- d) Find the equation of the chord of contact of tangent drawn to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the external point (x_1, y_1)
- e) Obtain the equation of the circle which cuts orthogonally each of the 3 circles
 $x^2 + y^2 = 16$, $x^2 + y^2 - 14x + 40 = 0$ and $x^2 + y^2 - 12y + 32 = 0$.
- f) Find the asymptote of the hyperbola $2x^2 - 5xy - 3y^2 - 5x - 3y - 21 = 0$.
- g) Find the condition that the line $lx + my + n = 0$ may represent a tangent line to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.
- h) Find the condition that the normals at the points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ may be concurrent.
- i) Develop the polar equation of a conic with a focus as the pole and line joining the focus to the corresponding vertex as the initial line.
- j) Prove that the semi-latus rectum of any conic (in polar) is the harmonic mean between the segments of any focal chord.

4. Write the answer of the following questions: **$6 \times 2 = 12$**

- a) Let (x, y) and (x', y') be the coordinate of a point referred to the original axes and new axes obtained by rotating an angle θ in the anti-clockwise direction. Develop the formulae to change (x, y) to (x', y') and vice versa.

Or

Define invariants. If by a rotational transformation about an origin the expression $ax^2 + 2hxy + by^2$ transforms to $a'x'^2 + 2h'x'y' + b'y'^2$ prove that $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$.

- b) Show that the equation $ab(x^2 + y^2) + (a^2 + b^2)xy + aby^2 + ab(a - b)(x - y) - a^2b^2 = 0$ represents two straight lines equidistant from the origin.

Or

Prove that the equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ represents a pair of parallel straight lines and find the distance between them.

5. Answer any two questions from the following: **$6 \times 2 = 12$**

- a) Show that the standard form of the conic $9x^2 + 24xy + 16y^2 - 126x + 82y - 59 = 0$ represents the equation of a parabola whose axis is the new x' -axis and whose latus rectum is of length 6.
- b) Prove that the general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents ellipse or hyperbola according to $ab - h^2$ as negative or positive.
- c) Define pole and polar. Find the pole of the straight line $lx + my + n = 0$ with respect to the circle $x^2 + y^2 = a^2$.

6. Answer the following questions: **$6 \times 2 = 12$**

- a) Define centre of a conic. Develop the formulae of the centre of the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

Or

Differentiate diameter and conjugate diameter of a conic. Find the condition that the pair of lines $Ax^2 + 2Hxy + By^2 = 0$ may be conjugate diameter of the conic $ax^2 + 2hxy + by^2 = 1$.

- b) If the straight line $r \cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e \cos\theta$. Prove that $(l \cos \alpha - ep)^2 + l^2 \sin^2 \alpha = p^2$.

Or

Prove that the shortest focal chord of conic is latus rectum.
