# DHANAMANJURI UNIVERSITY

## JUNE 2022

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| **Name of Programme** | **:** | **B.Sc Mathematics (Honours)** |
| **Semester** | **:** | **II** |
| **Paper Type** | **:** | **Core III** |
| **Paper Code** | **:** | **CMA-103** |
| **Paper Title** | **:** | **Real Analysis** |
| **Full Marks** | **:** | **100** |
| **Duration** | **:** | **3 Hours** |

*The figures in the margin indicate full marks for the questions. Answer all the questions.*

1. State the completeness property of $\mathbb{R}$. Prove that if $A$ and $B$ are non empty subsets of $\mathbb{R}$ that satisfy the property:$a \leq b$ for all $a \in A$ and all $b \in B$, then sub $A \leq \inf B$. 4

2. Show that the series $\frac{1,2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \cdots$ is convergent. 4

3. Prove that a sequence cannot converge to more than one limit. 5

4. What is an absolutely convergent series? Prove that every absolutely convergent series is convergent. 5

5. Prove that an upper bound $u$ of a non empty set $S\, in\, \mathbb{R}$ is the supremum of $S$ if and only if for every $\epsilon > 0$ there exists $s_\epsilon \in S$ such that $u - \epsilon < s_\epsilon$. 6

6. Prove the following: 6
   i) If $a \in \mathbb{R}$ and $a \neq 0$, then $a^2 > 0$.
   ii) $1 > 0$.
   iii) If $n \in \mathbb{N}$, then $n > 0$.

7. Prove that the following statements are equivalent: 6
   i) $S$ is a countable set.
   ii) There exists a surjection from $\mathbb{N}$ to $S$.
   iii) There exists a injection from $S$ to $\mathbb{N}$.

8. State and prove the Archimedean Property. 6

9. State and prove the Sandwich Theorem for limits. 6

10. Prove that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded. 6

11. State and prove Cantor's Theorem. 7

12. If $I_n = [a_n, b_n], n \in \mathbb{N}$, is a nested sequence of closed bounded intervals, then prove that there exists a number $\xi \in \mathbb{R}$ such that $\xi \in I_n$ for all $n \in \mathbb{N}$. 7

13. If $S$ is a subset of $\mathbb{R}$ that contains at least two points and has the property: if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$, then prove that $S$ is an interval. 8

Or

State and prove the Density Theorem for rational numbers.

14. State and prove the Bolzano-Weierstrass Theorem for sequences. 8

Or

State and prove Cauchy's General Principle of Convergence.

15. State and prove Leibnitz's Test for the convergence of an alternating series. 8

16. State and prove Cauchy's Root Test for the convergence of positive term series. 8