DHANAMANJURI UNIVERSITY

Examination, 2023 (Dec)

Four year course B.Sc. 1^{st} Semester

Name of Programme	**:**	**B.A/B.Sc. Mathematics (Honours)**
Semester	**:**	**I**
Paper Type	**:**	**Core-II (Theory)**
Paper Code	**:**	**CMA-102**
Paper Title	**:**	**Algebra**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions

All the questions.

UNIT-I

Choose any three questions

1. Answer the following questions: **6+4=10**

a) Prove that

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin\{\alpha+(n-1)\beta = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin(\alpha+\frac{n-1}{2}\beta)$$

b) Prove that

$$(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2(a^2+b^2)^{\frac{m}{2n}}\cos(\frac{m}{n}\tan^{-1}\frac{b}{a})$$

2. Answer the following questions: **7+3=10**

a) State and prove Gregory's series.

b) Prove that Log (-1)=(2n+1)iπ.

3. Answer the following questions: **7+3=10**

a) Prove that

$$\cos\alpha = 1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} - \frac{\alpha^6}{6!} + \cdots + (-1)^r\frac{\alpha^{2r}}{2r!} + \cdots infinity$$

b) Prove that $\tan\left\{i\log\frac{a-ib}{a+ib}\right\} = \frac{2ab}{a^2-b^2}$

4. Answer the following questions: 5+5=10

a) Find the sum of the series

$$\cos\theta - \frac{1}{2}\cos 2\theta + \frac{1}{3}\cos 3\theta - \cdots infinity \; (-\pi < \theta < \pi)$$

b) Using De Moivre's Theorem, solve $x^7 + x^4 + x^3 + 1 = 0$

5. Answer the following questions: 6+4=10

a) Prove that if $(a_1 + ib_1)(a_2 + ib_2) \ldots (a_n + ib_n) = A + iB$ then.

i) $\tan^{-1}\frac{b_1}{a_1} + \tan^{-1}\frac{b_2}{a_2} + \cdots + \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{B}{A}$

ii) $(a_1^2 + b_1^2)(a_2^2 + b_2^2) \ldots (a_n^2 + b_n^2) = A^2 + B^2$

b) Find the value of the series $1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \cdots infinity$

UNIT-II

Choose any three questions

6. Answer the following questions: 7+3=10

a) Prove that if a and b are positive and unequal then $\frac{a^m+b^m}{2} > (\frac{a+b}{2})^m$ except when m lies between 0 and 1.

b) Show that the equation $x^4 - 2x^3 - 1 = 0$ has at least two imaginary roots.

7. Answer the following question: 6+4=10

a) State and prove Holder's Inequality.

b) Solve $2x^3 + x^2 - 7x - 6 = 0$ given that the difference of two of the roots is 3.

8. Answer the following question: 5+5=10

a) If a,b,c are unequal, prove that $\frac{2}{a+b} + \frac{2}{b+c} + \frac{2}{c+a} > \frac{9}{a+b+c}$

b) Transform the equation into an equation lacking the second term $x^3 - 6x^2 + 4x - 7 = 0$.

9. Answer the following question: **7+3=10**

a) Solve the cubic equation $x^3 - 15x - 126 = 0$ by using Cardan's method.

b) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$.

10. Answer the following questions: **7+3=10**

a) Solve the biquadratic equation $x^4 + 6x^2 + 8x + 21 = 0$ by using Ferrari's method.

b) If a,b,c are unequal and positive , prove that

$$\frac{bc}{b+c} + \frac{ca}{c+a} + \frac{ab}{a+b} < \frac{a+b+c}{2}.$$

UNIT-III

Choose any two questions

11. Answer the following questions: **7+3=10**

a) Find the eigenvalues of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ and also find the Eigen vectors corresponding to the smallest eigenvalue.

b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.

12. Answer the following questions: **7+3=10**

a) Verify Cayley- Hamilton theorem for the square matrix A where A= $\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and hence find A^{-1}

b) Prove that the characteristic roots of a Hermitian matrix are real.

13. Answer the following questions: **7+3=10**

a) State and prove Cayley-Hamilton Theorem.

b) Determine the rank of the matrix $\begin{bmatrix} -2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$

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