DHANAMANJURI UNIVERSITY

Examination, 2023 (Dec)

Four year course B.Sc. 1st Semester

Name of Programme	**:**	**B.A/B.Sc. Mathematics (Honours)**
Semester	**:**	**I**
Paper Type	**:**	**Core-I (Theory)**
Paper Code	**:**	**CMA-101**
Paper Title	**:**	**Calculus**

Full Marks : 80

Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions

All the questions.

1. Choose the correct answers for each of the following questions: $\mathbf{1 \times 4 = 4}$

a) If y=log(x+a), then y_n is

i) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ ii) $\frac{(-1)^{n+1}(n+1)!}{(x+a)^n}$

iii) $\frac{(-1)^{n-1}(n-1)!}{(x+a)^n}$ iv) $\frac{(-1)^n(n-1)!}{(x+a)^{n-1}}$

b) If $x = rcos\theta, y = rsin\theta, then$ $\frac{\partial\theta}{\partial x}$ is equal to

i) $\frac{y}{x^2+y^2}$ ii) $\frac{-y}{x^2+y^2}$

iii) $\frac{x}{x^2+y^2}$ iv) $\frac{-x}{x^2+y^2}$

c) The whole area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

i) $\frac{1}{4}\pi ab$ ii) $\frac{1}{2}\pi ab$

iii) $\frac{1}{3}\pi ab$ iv) πab

d) The radius of curvature at any point for the curve

$x = a(cost + tsint), y = a(sint - tcost)$

i) a^2t ii) at^2

iii) at iv) $\sqrt{a}\, t$

2. Write very short answer for each of the following questions: $\mathbf{1 \times 10 = 10}$

a) If $y = Asin\, mx + Bcos\, mx, prove\ that\ y_2 + m^2y = 0.$

b) Find the value of c in the Mean Value Theorem

$f(a) - f(a) = (b - a)f'(c)\ if\ f(x) = x^2, a = 1, b = 2$

c) When is a function f(x,y) said to be homogeneous of degree n.

d) Find the radius of curvature at any point (s,Ψ) on the curve s=log sin Ψ.

e) Prove that $y = e^x$ is everywhere concave upwards.

f) Find the point on which the curve $y = x^3 - 3ax^2$ meets the $x - axis$.

g) Is the curve r=a(1+cosθ) symmetric about the initial line?

h) Evaluate: $\int_0^{\frac{\pi}{2}} sin^2xdx$

i) Write $\iint f(x, y)dxdy$ into polar co-ordinates.

j) Find the value of $\int_0^2 \int_{x^2}^{2-x} xydxdy$.

3. Write short answer for each of the following questions: $\mathbf{3 \times 10 = 30}$

a) If y = x^3 sin x, find y_n?

b) In the Mean value theorem f(a+h) =f(h) + hf'(a+θh), if a=1, h=3 and f(h) = $\sqrt{x}$, find θ.

c) Expand the function e^x in a finite series in powers of x, with the remainder in Cauchy's form in each case.

d) If u= $\sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ Show that x $\frac{\partial u}{\partial x}$ + y$\frac{\partial u}{\partial y}$ = 0.

e) State and prove Euler's theorem on homogeneous function of two variables.

f) Find the radius of the curvature of $y^2 = 4ax$ at any point(x,y).

g) Find the radius of the curvature of r = a(1- cos θ) at any point(r,θ).

h) Find the points of inflexion of the curve y=ae^{-8x^2}.

i) Find the length of an arc of the curve y= log secx from x=0 to $x = \frac{\pi}{3}$.

j) Evaluate $\iint xydxdy$ over the region in the positive quadrant in which $x + y \leq 1$.

4. Answer any two questions: **$6 \times 2 = 12$**

a) If y= $\sin^{-1} x$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - x^2y_n = 0$$

Find also the value of $(y_n)_0$

b) State and prove Lagrange's Mean value theorem.

c) Find the values of a,b,c so that $\lim_{x\to 0} \frac{ae^x - bcosx + ce^{-x}}{xsinx} = 2$.

5. Answer any two questions: **$6 \times 2 = 12$**

a) If u= log($x^3 + y^3 + z^3 - 3xyz$), then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{3}{(x+y+z)^3}$$

b) If $f_{xy}(x, y) = xy\frac{x^2 - y}{x^2 - y^2}$, when both x, y≠0, f(0,0)=0, show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

c) Find the maximum values of the function

$$f(x, y) = xy(a - x - y).$$

6. Answer any two questions: **$6 \times 2 = 12$**

a) Show that

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \int_0^{\frac{\pi}{2}} \cos^n x \, dx = \begin{cases} \frac{n-1}{n}, \frac{n-3}{n-2}, \frac{n-5}{n-4}, \ldots \ldots \ldots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \\ \frac{n-1}{n}, \frac{n-3}{n-2}, \frac{n-5}{n-4}, \ldots \ldots \ldots \frac{4}{5} \cdot \frac{2}{3} \cdot 1 \end{cases}$$

according as n is even or odd integer

b) Show that the area cut off a parabola $y^2 = 4ax$ by any double ordinates is $\frac{2}{3}$ of the corresponding rectangle contained by that double ordinate and its distance from the vertex.

c) Find the volume and surface area of the solid generated by revolving the cycloid x=a(θ+ sin θ) , y = a(1+ cos θ) about its base.

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